

Linear Equations: Number of Solutions

Linear equations can have **one solution**, **no solution**, or **infinitely many solutions**.

①

One Solution

If an equation is true for a single value of the variable, then it has exactly one solution.

Example

$$\begin{aligned} 5x - 4 &= 6 \\ 5x - 4 + 4 &= 6 + 4 \\ 5x &= 10 \\ \frac{5x}{5} &= \frac{10}{5} \\ x &= 2 \end{aligned}$$

The value $x = 2$ is the only value of x that makes the equation $5x - 4 = 6$ true.

②

No Solution

If an equation is false for all values of the variable, then it has no solution.

Example

$$\begin{aligned} 2c - c &= 5 + c \\ c &= 5 + c \\ c - c &= 5 + c - c \\ 0 &= 5 \end{aligned}$$

You get a false statement, $0 = 5$, when solving this equation. No matter what value is substituted for c , the equation $2c - c = 5 + c$ is false.

③

Infinitely Many Solutions

If an equation is true for all values of the variable, then it has infinitely many solutions.

Example

$$\begin{aligned} 3s - 2s &= s \\ s &= s \\ s - s &= s - s \\ 0 &= 0 \end{aligned}$$

You get a true statement, $0 = 0$, when solving this equation. No matter what value is substituted for s , the equation $3s - 2s = s$ is true.



Practice it! Determine if each equation has one solution, no solution, or infinitely many solutions. Write your answer on the blank.

$$6 + 9r = 9r + 6$$

$$2b - 5 = 9$$

$$-2q - 3 = -2q - 8$$

$$8p - 1 = 1 + 8p$$

$$4 + 7c = 11$$

$$-4n = -7n + 3n$$

$$6e - 10 = 10 - 2e + 10e$$

$$4(x - 5) = 4x - 20$$

$$2(3g + 1) = 3g + 1 + 3g$$

$$8 + 5(f - 2) = 2f - 2 + 3f$$

$$-8 - 4p = 5(p + 2)$$

$$8m - 4 - 2m = 3(2m - 1)$$
