Systems of Equations: Number of Solutions

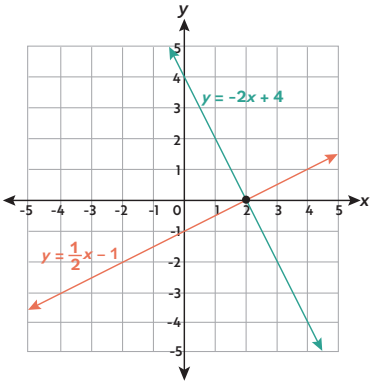
A **solution** to a system of linear equations is a point where the lines intersect. Systems of linear equations can have one solution, no solution, or infinitely many solutions.

If the lines intersect at exactly one point, then the system has **one solution**. This will happen if the slopes of the two lines are different.

For example, consider this system of equations:

$y = -2x + 4$

$y = \frac{1}{2}x - 1$



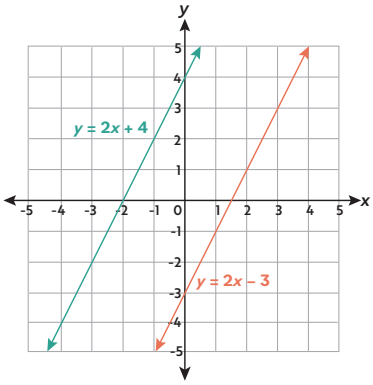
Since the slopes of the lines are different, the lines intersect at one point. So, this system has one solution.

If the lines never intersect, then the system has **no solution**. This will happen if the lines are parallel. Parallel lines have the same slope but different y-intercepts.

For example, consider this system of equations:

$y = 2x + 4$

$y = 2x - 3$



Since the slope of the lines are the same and the y-intercepts are different, the lines are parallel. So, this system has no solution.

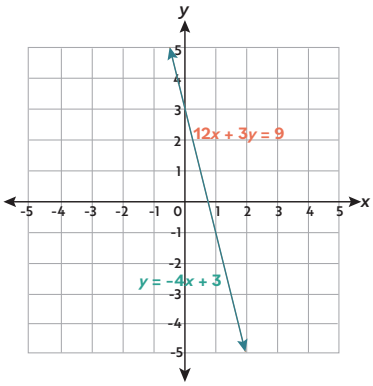
If the lines intersect at every point, then the system has **infinitely many solutions.** This will happen if the equations in the system represent the same line.

For example, consider this system of equations:

$y = -4x + 3$
$12x + 3y = 9$

Solve the second equation for y to find the slope and y-intercept.

$12x + 3y = 9$
$3y = -12x + 9$
$y = -4x + 3$



Since the equations represent the same line, this system has infinitely many solutions.

Practice! Determine if each system has one solution, no solution, or infinite solutions. Circle your answer. Then explain how you know. Explanations may vary.

$y = 3x + 2$
$y = 2x + 2$

(One solution) / No solution / Infinite solutions

Explain: The slopes of the lines are 3 and 2. Since the slopes are different, the lines intersect at one point.

$y = 5x + 4$
$y = 5x - 2$

One solution / (No solution) / Infinite solutions

Explain: Both lines have a slope of 5, but the lines have different y-intercepts. So, the lines are parallel.

Systems of Equations: Number of Solutions

Keep going! Determine if each system has one solution, no solution, or infinite solutions. Circle your answer. Then explain how you know. Explanations may vary.

$y = -4x + 1$
$y = 1 - 4x$

One solution
No solution
(Infinite solutions)

Explain: Both equations are equivalent to $y = -4x + 1$. So, the lines intersect at every point.

$4x - y = 2$
$y = x - 8$

(One solution)
No solution
Infinite solutions

Explain: The slopes of the lines are 4 and 1. Since the slopes are different, the lines intersect at one point.

$y = -2x + 5$
$6x + 3y = 15$

One solution
No solution
(Infinite solutions)

Explain: Both equations are equivalent to $y = -2x + 5$. So, the lines intersect at every point.

$2x + y = 7$
$3y = -6x + 9$

One solution
(No solution)
Infinite solutions

Explain: Both lines have a slope of -2, but the lines have different y-intercepts. So, the lines are parallel.

$y = 1.2x - 0.2$
$y = 3.8x - 0.2$

(One solution)
No solution
Infinite solutions

Explain: The slopes of the equations are 1.2 and 3.8. Since the slopes are different, the lines intersect at one point.

$-\frac{7}{2}x + \frac{1}{2}y = 4$
$y = 7x + 8$

One solution
No solution
(Infinite solutions)

Explain: Both equations are equivalent to $y = 7x + 8$. So, the lines intersect at every point.

$24x + 8y = 3.2$
$y = -3x + 0.2$

One solution
(No solution)
Infinite solutions

Explain: Both lines have a slope of -3, but the lines have different y-intercepts. So, the lines are parallel.

$6y = 9x + 1.8$
$1.5x - y = -0.3$

One solution
No solution
(Infinite solutions)

Explain: Both equations are equivalent to $y = 1.5x + 0.3$. So, the lines intersect at every point.