Linear Equations: Number of Solutions

Linear equations can have **one solution**, **no solution**, or **infinitely many solutions**.

One Solution

If an equation is true for a single value of the variable, then it has exactly one solution.

Example

$5x - 4 = 6$

$5x - 4 \textbf{ + 4} = 6 \textbf{ + 4}$

$5x = 10$

$\dfrac{5x}{\textbf{5}} = \dfrac{10}{\textbf{5}}$

$x = 2$

The value $x = 2$ is the only value of x that makes the equation $5x - 4 = 6$ true.

No Solution

If an equation is false for all values of the variable, then it has no solution.

Example

$2c - c = 5 + c$

$c = 5 + c$

$c \textbf{ - c} = 5 + c \textbf{ - c}$

$0 = 5$

You get a false statement, $0 = 5$, when solving this equation. No matter what value is substituted for c, the equation $2c - c = 5 + c$ is false.

Infinitely Many Solutions

If an equation is true for all values of the variable, then it has infinitely many solutions.

Example

$3s - 2s = s$

$s = s$

$s \textbf{ - s} = s \textbf{ - s}$

$0 = 0$

You get a true statement, $0 = 0$, when solving this equation. No matter what value is substituted for s, the equation $3s - 2s = s$ is true.

Practice it! Determine if each equation has one solution, no solution, or infinitely many solutions. Write your answer on the blank.

$6 + 9r = 9r + 6$	$2b - 5 = 9$	$-2q - 3 = -2q - 8$
infinitely many solutions	one solution, $b = 7$	no solution
$8p - 1 = 1 + 8p$	$4 + 7c = 11$	$-4n = -7n + 3n$
no solution	one solution, $c = 1$	infinitely many solutions
$6e - 10 = 10 - 2e + 10e$	$4(x - 5) = 4x - 20$	$2(3g + 1) = 3g + 1 + 3g$
one solution, $e = -10$	infinitely many solutions	no solution
$8 + 5(f - 2) = 2f - 2 + 3f$	$-8 - 4p = 5(p + 2)$	$8m - 4 - 2m = 3(2m - 1)$
infinitely many solutions	one solution, $p = -2$	no solution